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1 Word Problems: Ratios

A ratio is nothing more than a relationship between numbers, just like a fraction, a decimal, or a percent. A ratio is often represented with a colon (":"), as in the ratio 3 : 2, but that ratio is equivalent to several other numbers you may be more comfortable working with. For example, 3 : 2 is equivalent to the fraction $\frac{3}{2}$. A ratio can also be expressed as a percent: 1 : 4 is equivalent to $\frac{1}{4}$, which equals 25%. Finally, a ratio can also be expressed as a decimal. First, you would convert to a fraction or percent, then convert to a decimal. (That will rarely be useful, however.)

One of the most common types of ratios you'll encounter is the "part-to-part" ratio. When the two parts of a ratio do not overlap, they are a part to part ratio. For example: if the ratio of boys to girls in the first grade is 4 : 5, there's no overlap between boys and girls; each one of those segments represents a part of the whole class.

Further, there can be more than two parts in a part to part ratio. On the GRE, however, you'll only occasionally see a ratio with more than three parts. An example of a three-part ratio would go like this: "The ratio of Stockholder A's holdings to Stockholder B's holdings to Stockholder C's holdings is 5 : 2 : 1." Again, there's no overlap between any of the individual holdings.

The other common type of ratio is a "part-to-whole" ratio. A part to whole ratio is more like a fraction. You can usually think of fractions as "part over whole," so a ratio expressing the relationship between a part and a whole is very similar to that. Here's what that might look like:

> "The relationship between the number of teenage members of a club and the number of total members is expressed by the ratio 2 : 5."

In this case, the number of teenage members is a subset of the total number of members.

Everything we've discussed so far is very elementary. GRE questions start getting tricker when you have to convert from one type of ratio to another. Whenever you have a part to part ratio, you can convert it into a part to whole ratio.

Consider the first example above: "The ratio of boys to girls in the first grade is 4 : 5." If, for every 4 boys, there are 5 girls, then for every 4 boys , there are a total of 9 children. Thus, the ratio of boys to girls generates two more ratios:

boys to total: 4 : 9
girls to total: 5 : 9

Similarly, if you have a part to whole ratio, you can convert it into a part to part ratio. Again, to use an example from above: "The relationship between the number of teenage members of a club and the number of total members is expressed by the ratio 2 : 5."

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If, for every 5 members, 2 are teenagers, that means 3 are not teenagers. Thus, this part to whole ratio generates two other ratios:

non-teenagers to total: 3 : 5

non-teenagers to teenagers: 3 : 2

One last thing: note that any of these ratios could be reversed: to say that the ratio of non-teenagers to teenagers is 3 : 2 is equivalent to saying that the ratio of teenagers to non-teenagers is 2 : 3.

Still, we haven't quite gotten to where GRE questions will expect you to be. It's a rare question that asks explicitly for a ratio; more commonly, you'll be required to use a ratio to find a number.

In order to find a number from a ratio, you need some other number. If I just tell you the relationship between two quantities, you can never find the actual amount of one of the quantities without further information. If, for instance, the ratio of native Spanish speakers to native English speakers in a certain group is 3 : 2, the group could consist of 3 Spanish speakers and 2 English speakers or 300 Spanish speakers and 200 English speakers. (Or an infinite number of other possibilities!)

Thus, the important piece of data in such a question is an actual number. Given the ratio above, let's say you're told that there are 50 people in the group and you need to determine the number of native English speakers. If you're given the total and need one part, you'll need a part to whole ratio to answer the question.

The equation you'll set to answer such a question will almost always look like this:

$$\frac{part(ratio)}{whole(ratio)} = \frac{part(actual)}{whole(actual)}$$

In this case, the ratio of English speakers (part) to total group members (whole) is 2 : 5, so the left side of the equation is $\frac{2}{5}$. The actual number of group members is 50 and the number of English speakers is our unknown, so the right side of the equation is $\frac{x}{50}$. Put it all together, and the equation looks like this:

$$\frac{2}{5} = \frac{x}{50}$$

To solve, cross-multiply and isolate x:

$2(50) = 5x$

$x = \frac{100}{5} = 20$

There is one thing worth accentuating about that process. It doesn't matter what type of ratio you're given. In this case, we started with a part to part ratio. What matters is the actual numbers we're given and the one we need. Since we were provided the number of total group members and we needed the number of English speakers, the ratio we needed was the one between those two quantities.

Most of the time, when ratios arise in GRE problems, you'll do something like what we've discussed so far. But sometimes, you'll be asked to manipulate a ratio. In the simplest form, a question will tell you that a ratio "doubles" or

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"halves." In those cases, it's easier to think of the ratio as a fraction. So, if the ratio of boys to girls is initially $2:3$, and then it doubles, just multiply the ratio/fraction by two:

$$\frac{2}{3}(2) = \frac{4}{3} = 4:3$$

It's more complicated when you're asked to add to or subtract from a ratio. Let's say, again, that the ratio of boys to girls is $2:3$. If two more boys are added, what is the resulting ratio? The short answer is that we don't know. In order to know the effect of two boys, we need to know how many children were there in the first place, and a ratio doesn't give us that.

However, we can express the result algebraically. If the ratio is $2:3$, we typically write is as a fraction like so:

$$\frac{2}{3}$$

But, if we want to represent the actual number of boys and girls, we can add a variable. You might think of it as a multiplier:

$$\frac{2x}{3x}$$

The relationship is the same: the x's cancel out. But now, each part of the ratio represents an actual number. If there are 20 boys, $x = 10$ and $\frac{2x}{3x}$ is the same as $\frac{20}{30}$. If there are 6 boys, $x = 3$ and $\frac{2x}{3x} = \frac{6}{9}$. No matter what the multiplier is (how many children there actually are), $\frac{2x}{3x}$ represents that number.

So, if you want to add two boys to the group, add it to the fraction that represents the original number (not just the ratio) of children:

$$\frac{2x+2}{3x}$$

It may not be very satisfying right away, but it comes in handy. If this were an actual GRE question, it would give you the original ratio of boys to girls, tell you that two more boys were added, and then tell you that the resulting ratio was, say, $4:5$. Thus, you know that the ratio you came up with, $\frac{2x+2}{3x}$, is equal to $\frac{4}{5}$, and you can solve for the value of x. Once you have x, you can determine the the original number of boys, the original number of girls, or any number of other things.